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High-Efficiency Modes in Avalanche Diodes

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ABSTRACT

Some recent experimental results on a high-efficiency
mode in avalanche diodes are not explained by the results to
date of TRAPATT mode theory despite careful and compre-
hensive modeling. This note explores one aspect of large-
signal theory not previously examined in the literature, sug-
gesting one reason for the above situation. An explicit third
boundary condition (in addition to the conventional two boundary
conditions) involving the diode-circuit interaction is presented
which does not appear in the prior literature.

I. Introduction

Recent repeatable experiments by Kostishack[1] have demonstrated in several different measurement systems efficiencies up to 75% with avalanche silicon diodes. These results apparently are not explained[1] by recent analyses [2, 3, 4] nor the careful and extensive computer simulations [5, 6] to date of the TRAPATT mode which have not yet revealed a maximum achievable efficiency of greater than about 60% for silicon avalanche diodes [5, 6, 7]. This situation demonstrates that the presently used theoretical methods have not systematically exhausted the physically interesting solutions for the high efficiency avalanche diode mode. This note examines the basic large-signal theory and finds that a third boundary condition (to supplement the usual two boundary conditions) describing the diode-circuit interaction is needed for completeness. Since the high-efficiency mode requires critical circuit tuning one might anticipate the existence of the above constraint. On the other hand, one must explain how presently available theories have been utilized without invoking the above boundary condition. This note will briefly consider the above ideas.

II. Basic Equations

Consider first the usual equations for electron and hole current continuity.

$$\partial n/\partial t = -(1/q)\, \partial J_n/\partial x + g \tag{1}$$

$$\partial p/\partial t = (1/q)\, \partial J_p/\partial x + g \tag{2}$$

where $J_n = -q \left| v_n \right| n$, $J_p = -q \left| v_p \right| p$, v_n and v_p are constant and the symbols have their usual meaning. Before proceeding, we observe that the above are two coupled differential equations involving the three unknowns n, p and E. A third independent equation, which cannot be derived from Eqs. (1) and (2), must be used to supplement the above. Generally, Poisson's equation

$$\partial E/\partial x = (q/\epsilon)\,(p - n) \tag{3}$$

is used to augment Eqs. (1) and (2), yielding a system of three coupled equations in three unknowns.

Equations (1), (2) and (3) can be used to derive the equation for total current

$$\partial J_t/\partial x = 0 \tag{4}$$

where $J_t(t) \equiv J_n + J_p + \epsilon\,\partial E/\partial t$. Conversely, Eqs. (1), (2) and (4) can be used to derive (a) Eq. (3) in the small-signal case, and (b) Eq. (3) with an unknown added constant of integration in the large-signal case. However, one performs the equation count (i.e., with Eqs. (1), (2) and (3), or Eqs. (1), (2) and (4), the number of required equations is never less than three.

III.   The Small-Signal Case

In small-signal theory there are several equivalent formulations which may be used to reach the desired result. This tends to obscure the underlying nature of the problem. One such set of equivalent formulations will be exhibited in this section, where it will be shown that the small-signal impedance may be computed without use or knowledge of a third

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boundary condition.

Combining Eqs. (1), (2) and (3) one may obtain a second-order, inhomogeneous, differential equation

$$\left[\frac{\partial^2}{\partial x^2} + k_m^2\right] E_1 = \frac{J_{t1}}{\epsilon v} (2\alpha_o - j\omega/v) \tag{5}$$

where $k_m^2 = (\omega/v)^2 + 2j\alpha_o (\omega/v) - 2\alpha_o' J_o/\epsilon v$, α_o is α evaluated at E_o (the dc value of E), $\alpha_o' \equiv d\alpha/dE$ evaluated at $E = E_o$, and $\alpha = \beta$, and $v_n = v_p = v$ = constant have been assumed for simplicity. E_1 and J_{t1} are the time-harmonic components of E and J. Alternatively, by a different manipulation of Eqs. (1), (2) and (3), or direct differentiation of Eq. (5), one obtains

$$\frac{\partial}{\partial x} \left[\frac{\partial^2}{\partial x^2} + k_m^2 \right] E_1 = 0 \tag{6}$$

which is a third-order homogeneous differential equation. The solution to both equations is

$$E = \sum_{i=1}^{3} \beta_i e^{-jk_i x} = \beta_1 \left[1 + \left(\frac{\beta_2}{\beta_1}\right) e^{-jk_m x} + \left(\frac{\beta_3}{\beta_1}\right) e^{+jk_m x} \right] \tag{7}$$

with three unknown constants β_1, β_2 and β_3. The constant β_1 may be expressed in terms of J_{t1} through the result $\beta_1 = 2\alpha_o - j\omega/v) J_{t1}/\epsilon v k_m^2$. However, β_1 remains an unknown until all boundary conditions are applied because J_{t1} is an unknown constant.

The standard boundary conditions for the avalanche region of an

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avalanche diode are $J_n = 0$ at $x = -L/2$ and $J_p = 0$ at $x = L/2$. It follows that $\beta_3 = \beta_2$ and β_3/β_1 is a specific calculable function of the parameters, but that β_1 is as yet undetermined. In small-signal theory the main parameter of significance is the impedance $Z = V_1/J_{t1} A$ (where $V = - \int E dx$ is the diode voltage, and A is the cross-sectional area of the diode) which may be computed without ambiguity beacuse β_1 is proportional to J_{t1} and so β_1 (or J_{t1}) cancels out of the ratio for Z.

IV.   A Large-Signal Case

In large-signal theory the same two-boundary conditions apply and Bartelink and Scharfetter[3] have derived (from Eqs. (1), (2) and (3)) a partial differential equation

$$\epsilon \frac{\partial}{\partial x} \left[V(E) \frac{\partial E}{\partial x} \right] = \left[2\alpha(E) - \frac{\partial}{\partial t} \frac{1}{V(E)} \right] \left[J_t(t) - \epsilon \frac{\partial E}{\partial t} \right] \quad (8)$$

which is second order when $J_t(t)$ is assumed known. Eq. (8) reduces to Eq. (7) in the small-signal case. The problem, however, is that $J_t(t) \equiv J_n + J_p + \epsilon \partial E/\partial t$ is an unknown function for which the time-dependence would, ideally, be determined at the end of the problem solving. Clever guesses at $J_t(t)$ apparently have not systematically exhausted the physically interesting solutions of the basic equations as Kostishack's experiments now show[1], and the output power and efficiency are dependent upon the function $J_t(t)$ guessed. Typical choices for $J_t(t)$ in computer simulations and analyses are a constant step in current (for which no ac power is extracted from the device[2]) and the square-wave (which can lead about to 60% efficiencies).

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Alternatively theorists have simplified the basic system of equations by the use of various approximations such as assuming $J_n + J_p$ (and hence E) to be independent of x in the avalanche region. Such approximations have helped in obtaining useful approximate solutions for the large-signal IMPATT case, however no indication of the high-efficiency results mentioned earlier have been revealed by these analyses.

V.   A Formulation

A description of the diode-circuit interaction may be added as a third, albeit complicated, boundary condition. Assume that the diode directly feeds a transmission line. On the transmission line one has the incident fields from the bias source plus reflections created by the diode. This may be written in the customary circuit formulation $I_c = I_s \left[1 - \Gamma \right]$ where I_c is the circuit current at the diode terminals, I_s is the applied bias current, Γ is the reflection coefficient seen at the diode terminals and is given by $\Gamma = \left[V_d - Z_o I_c \right] / \left[V_d + Z_o I_c \right]$, V_d is the total large-signal voltage across the diode terminals, and Z_o is the impedance of the transmission line containing the diode as a series element plus the usual series of slug tuners. Because the microwave circuit as seen from the diode terminals is a linear system, superposition may be applied. The circuit current equals the total diode current, i.e., $I_c = I_t$ which is independent of position x in the diode. For convenience I_t may be evaluated at one of the diode boundaries, say x = -L/2. Then $I_t = A \{ J_{p2} + \epsilon \, \partial E_2 / \partial t \}$ where J_{p2}, $\partial E_2 / \partial_t$, and $J_n = 0$ are the indicated variables evaluated at x = -L/2. The result is a

fairly messy (third) boundary condition, but one which may be evaluated and utilized in an iterative fashion.

As an illustration imagine that the time axis was divided into "sufficiently" small intervals. Suppose at $t = t_o$ the source generated wave first impinges on and is reflected from the diode. At the initial instant the diode is inert and its reflection coefficient in combination with the assorted slug tuners, Γ_o, may be calculated. If the diode current I_{t_o} (at $t - t_o$) is sufficiently high, it creates space-charge effects and will initiate activity. Approximating Γ_1 at time t_1 as Γ_o, I'_{t1} and V'_{d1} are calculated approximately and hence $\Gamma'_1 = \left[V'_{d1} - Z_o I'_{t1} \right] / \left[V'_d + Z_o I'_{t1} \right]$ which leads in turn to the refined result, V''_{d1} and I''_{t1}. This in turn leads to a refined value of

$$\Gamma''_1 = \left[V''_{d1} - Z_o I''_{t1} \right] / \left[V''_{d1} - Z_o I''_{t1} \right] .$$ The process is repeated until little change in Γ_1, is noted, then the calculations proceed to $t = t_2$ where the sequence is repeated starting with the approximation $\Gamma_2 = \Gamma_1$.

In summary a procedure has been discussed which makes use of a third boundary condition in the analysis of high-efficiency avalanche diodes. At the time of conclusion of the present work, a paper[8] on the same topic has appeared in the literature, but it does not formulate the problem as proposed here, nor account for the recent high efficiency results of Kostishack[1].

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